Announcement: Next week Office Hours

M 2-3 in LSK 300

Th 11-12:30 in LSK 300

Will book one more.

- Class survey | evaluation

Recap: Last time: - Derivative of sin & cos

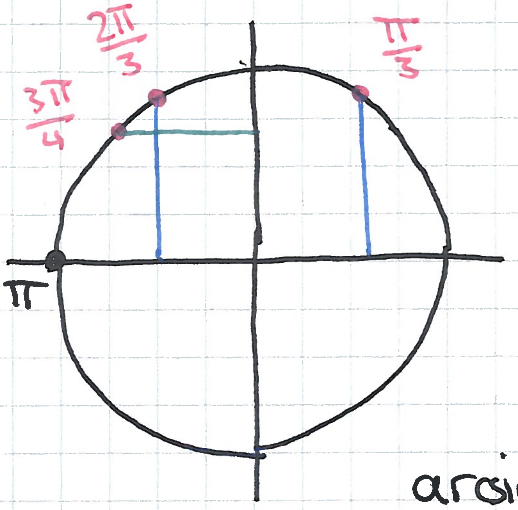
$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' \xrightarrow{\text{Quotient Rule}} \frac{1}{\cos^2 x} = \sec^2 x$$

Today: - Inverse trig arcsin, arccos, arctan } domain, range.

- Applications
- Their derivatives



$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

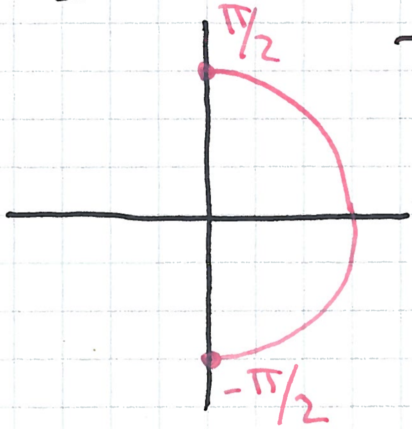
$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$\arcsin(x) = \theta \iff$ "What angle θ gives $\sin\theta = x$?"

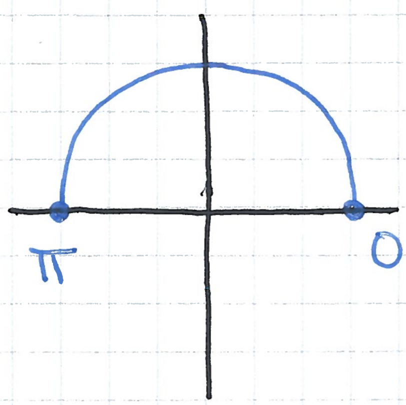
Ex: $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$, or $\frac{2\pi}{3}$?

In order to define arcsin, we must make a choice.

Declare: $\boxed{\arcsin(x)}$ has range $\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$



This encompasses all possible values of $\sin\theta$.

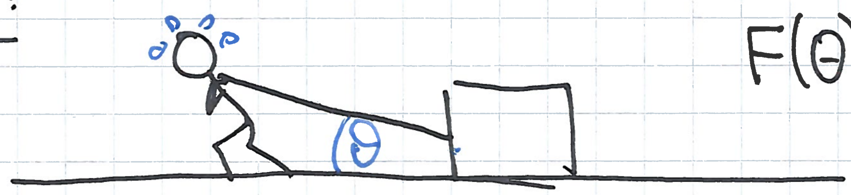


$\boxed{\arccos(x)}$

range $\boxed{[0, \pi]}$

Both have domain $[-1, 1]$.

Q2:



$$F(\theta) = \frac{50}{\cos\theta + \mu\sin\theta}$$

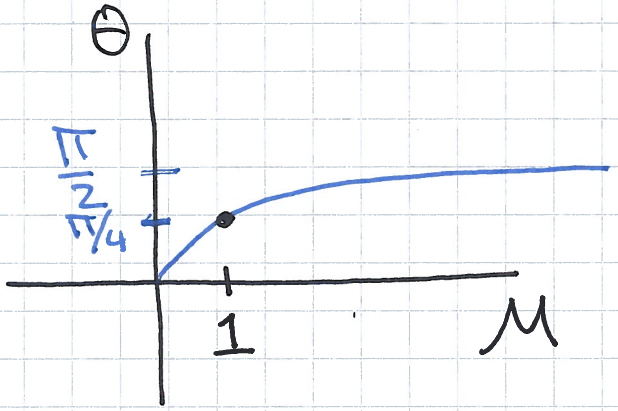
- θ larger $\rightsquigarrow$ less friction

Minimize $F(\theta)$

$\Updownarrow$

Maximize $\cos\theta + \mu\sin\theta$.

$$(\cos\theta + \mu\sin\theta)' = -\sin\theta + \mu\cos\theta = 0$$



$$\mu\cos\theta = \sin\theta$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

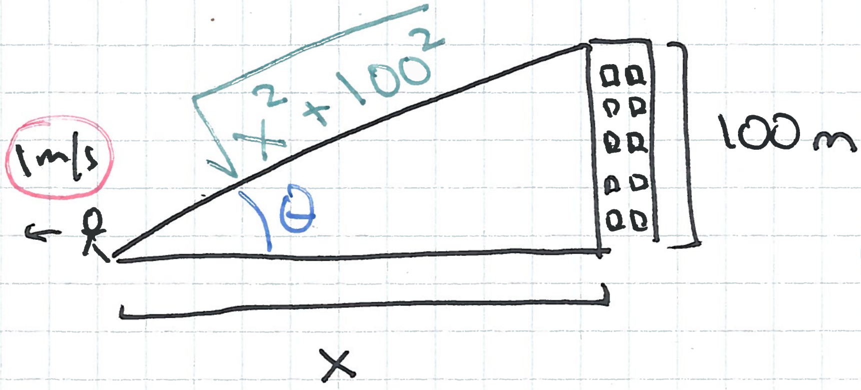
$$\boxed{\theta = \arctan(\mu)}$$

Note: $\cos\theta + \mu\sin\theta = A\sin(\theta + P)$

or $A\cos(\theta + P)$ for some A amplitude, P phase

Q: How to find P in terms of μ? (Challenge!)

Ex: when $\mu = 1$, $A\cos(\theta - \frac{\pi}{4})$



θ = viewing angle

= "How big it looks"

$\frac{dx}{dt} = 1\ m/s$ Q: What's $\frac{d\theta}{dt}$?

$\operatorname{arccot}(x)' = -\frac{1}{1+x^2}$

same thing $\arctan(x)' = \frac{1}{1+x^2}$

$\arcsin(x)' = \frac{1}{\sqrt{1-x^2}}$

⋮

- Relate x & θ.

$$\boxed{\tan\theta = \frac{100}{x}}$$

} Derivative both sides with respect to t

$\left.\begin{matrix}\sin\theta \\ \cos\theta\end{matrix}\right\}$ will get messy.

$\left(\tan \xrightarrow{\text{der.}} \sec^2\right)$

$$\theta' \sec^2\theta = -\frac{100}{x^2} \cdot \underbrace{x'}_{=1}$$

$$\theta' = \frac{1}{\sec^2\theta} \cdot \left(-\frac{100}{x^2}\right) = -\frac{100\cos^2\theta}{x^2} = -\frac{100 \cdot x^2}{x^2(x^2+100^2)}$$

But $\cos\theta = \frac{x}{\sqrt{x^2+100^2}}$

$$= \boxed{-\frac{100}{x^2+100^2}}$$